lgonzalz@vxcern.cern.ch

February 1997

SPACE, TIME AND SUPERLUMINAL PARTICLES

L. GONZALEZ-MESTRES

Laboratoire de Physique Corpusculaire, Collège de France
11 pl. Marcellin-Berthelot, 75231 Paris Cedex 05 , France

and

Laboratoire d'Annecy-le-Vieux de Physique des Particules
B.P. 110 , 74941 Annecy-le-Vieux Cedex, France

Abstract

If textbook Lorentz invariance is actually a property of the equations describing a sector of matter above some critical distance scale, several sectors of matter with different critical speeds in vacuum can coexist and an absolute rest frame (the vacuum rest frame, possibly related to the local rest frame of the expanding Universe) may exist without contradicting the apparent Lorentz invariance felt by "ordinary" particles (particles with critical speed in vacuum equal to c , the speed of light). The real geometry of space-time will then be different from standard Lorentz invariance, and the Poincaré relativity principle will be a local (in space and time), approximate sectorial property. It seems natural to assume that particles with critical speed in vacuum different from c are superluminal.

We illustrate such a scenario using as an example a spinorial space-time where the modulus of the spinor, associated to the time variable, is the size of an expanding Universe. Several properties of superluminal particles, and of matter without a universal relativity principle, are discussed in view of experimental applications. If the vacuum rest frame is close to that suggested by the cosmic microwave background, experimental searches for superluminal particles on earth should mainly contemplate a laboratory speed range around 10^3 c , even for very high energy superluminal cosmic rays. The detectability of several consequences of the new scenario is briefly discussed.

1. INTRODUCTION

In recent papers [1-5] , we pointed out that the apparent Lorentz invariance of the laws of physics does not imply that space-time is indeed minkowskian. Lorentz invariance can be just a property of the equations describing a sector of matter above some critical distance scale. Then, an absolute rest frame (possibly related to the local rest frame of the expanding Universe) may exist without contradicting the apparent Lorentz invariance felt by the particles we are made of. We suggested the possible existence of superluminal sectors of matter, i.e. of particles with positive mass and energy but with a critical speed in vacuum much higher than the speed of light c . Sectors of matter with different critical speeds in vacuum may coexist, just as in a perfectly transparent crystal it is possible to identify at least two critical speeds: those of sound and light. Interaction between the "ordinary" sector, i.e. particles with critical speed in vacuum equal to c , and the superluminal sectors, would break Lorentz invariance. But such interactions would be expected to be basically very high-energy, short distance phenomena not incompatible with successful conventional tets of Lorentz invariance. Several important physical and cosmological implications were discussed in a scenario with several critical speeds for particles in vacuum.

Admitting the possible existence of several sectors of matter with different critical speeds in vacuum, some stringent form of grand-unified or primordial constituent background seems necessary in order to explain why so many different particles (quarks, leptons, gauge bosons...) have the same critical speed. This is in agreement with present ideas in particle theory, and to some extent improves their formulation as it appears that grand unification and universality of the critical speed in vacuum may be expressions of a single symmetry inside each sector. Our hypothesis extends the current approach, but does not really contradict its basic philosophy. However, the superluminal particles we propose are definitely not space-like states of the ordinary ones: they are a new kind of matter related to new degrees of freedom not yet discovered experimentally. If superluminal particles exist, they may considerably modify the Big Bang scenario, provide most of the cosmic (dark) matter, lead the evolution of the Universe, influence low-energy physics and be produced at very high-energy accelerators (e.g. LHC) or found in experiments devoted to high-energy cosmic rays (e.g. AMANDA [6]) where they can yield very specific signatures allowing to detect events at extremely small rates (e.g. [5] , but see also Sect. 4).

In this note, we would like to discuss some possible implications of the new approach for our description of space, time and elementary particles. As an example, a previously considered $SU(2)$ spinorial space-time [4 , 5] will be chosen as the framework. If the structure of space-time reflects basically the properties of matter at the scales under consideration, it should to some extent account for the structure and evolution of the Universe, as well as for phenomena like spin 1/2 which cannot be described in a natural way using conventional space-time coordinates. In general relativity, the gravitational properties of matter modify the local metric of space-time, but a much closer connection between matter and space-time can be imagined incorporating deeper dynamical properties. Also, departure from the Poincaré relativity principle yields new fundamental physics (new particles and interactions, motion "backward in time" in some frames...), as will be discussed later.

2. THE SPINORIAL SPACE-TIME

Instead of four real numbers, we take space-time to be described by two complex numbers, the components of a $SU(2)$ spinor. From a spinor ξ , it is possible to extract a $SU(2)$ scalar, $\mid \xi \mid^2 = \xi^\dagger \xi$ (where the dagger stands for hermitic conjugate), and a vector $\vec{\mathbf{z}} = \xi^\dagger \vec{\sigma} \xi$, where $\vec{\sigma}$ is the vector formed by the Pauli matrices. In our previous papers on the subject [4 , 5] , we proposed to interpret $t = \mid \xi \mid$ as the time. If the spinor coordinates are complex numbers, one has: $z = t^2$ where z is the modulus of $\vec{\mathbf{z}}$. It does not seem possible to interpret $\vec{\mathbf{z}}$ as providing the space coordinates: one coordinate, corresponding to an overall phase of the spinor, is missed by t and $\vec{\mathbf{z}}$. Therefore, a different description of space seems necessary in this approach.

Interpreting t as the time has at first sight the drawback of positive-definiteness and breaking of time reversal, but this can be turned into an advantage if t is interpreted as an absolute, cosmic time (geometrically expanding Universe). An arrow of time is then naturally set, and space-time geometry incorporates the physical phenomenon of an expanding Universe. As space translations and rotations are by definition transformations leaving time invariant, the space coordinates should be built by considering the polar coordinates in the hypersphere (i.e. the spherical hypersurface) of constant time, $\mid \xi \mid = t_0$ where t_0 is a value of time [4 , 5]. On this hypersphere, a point ξ can be decribed as:

$$\xi \;=\; U \xi_0 \tag{1}$$

where U is a $SU(2)$ transformation and ξ_0 a constant spinor (hereafter identified with the observer position) on the sphere $t = t_0$. Writing:

$$U \;=\; exp\ (i/2\ \ t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{x}}) \;\equiv U(\vec{\mathbf{x}}) \tag{2}$$

the vector $\vec{\mathbf{x}}$, with $0 \ \leq x$ (modulus of $\vec{\mathbf{x}}$) $\leq 2\pi t_0$, can be interpreted as the position vector at constant time t_0 . It is unique, except for a 2π rotation ($x \;=\; 2\pi t_0$, $U \;=\; -1$). The natural metric is $dr^2(\vec{\mathbf{x}}\ ,\ \vec{\mathbf{dx}}) \;=\; (\vec{\mathbf{dx}}^*)^2$, where $\vec{\mathbf{dx}}^*$ is defined by:

$$U(\vec{\mathbf{x}}\ +\ \vec{\mathbf{dx}}) \;\;=\;\; U(\vec{\mathbf{dx}}^*)\ U(\vec{\mathbf{x}}) \tag{3}$$

or, in terms of exponentials:

$$exp\ [i/2\ \ t_0^{-1}\ \vec{\sigma}.(\vec{\mathbf{x}} + \vec{\mathbf{dx}})] \;\;=\;\; exp\ (i/2\ \ t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{dx}}^*)\ exp\ (i/2\ \ t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{x}}) \tag{4}$$

leading at infinitesimal level to:

$$\vec{\mathbf{dx}}^* \;=\; \vec{\mathbf{dx}}_{\mathbf{L}}\ +\ 2t_0\ [cos\ (t_0^{-1}x/2)\ sin\ (t_0^{-1}x/2)\ x^{-1}\ \vec{\mathbf{dx}}_\perp\ -\ sin^2\ (t_0^{-1}x/2)\ x^{-2}\ \vec{\mathbf{x}} \wedge \vec{\mathbf{dx}}] \tag{5}$$

where $\vec{\mathbf{dx}}_{\mathbf{L}} \;=\; x^{-2}\ (\vec{\mathbf{x}}.\vec{\mathbf{dx}})\ \vec{\mathbf{x}}$ and $\vec{\mathbf{dx}}_\perp \;=\; \vec{\mathbf{dx}} - \vec{\mathbf{dx}}_{\mathbf{L}}$. From (5), we consistently recover $\vec{\mathbf{dx}}^*(x = 0) \;=\; \vec{\mathbf{dx}}$, whereas $\vec{\mathbf{dx}}^*(x = 2\pi t_0) \;=\; \vec{\mathbf{dx}}_{\mathbf{L}}$. Any closed path of the form: $U(\alpha \vec{\mathbf{u}}) \;=\; exp\ (i/2\ \ \alpha t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{u}})$ from $\alpha = -2\pi t_0$ to $\alpha = 2\pi t_0$, where $\vec{\mathbf{u}}$ is a unitary three-dimensional real vector, defines a geodesic on the hypersphere and on $SU(2)$. The local volume element at ξ is $d^3\vec{\mathbf{x}}^* \;=\; dx_1^* dx_2^* dx_3^*$.

It is obvious that, under a $SU(2)$ transformation V , U transforms into VUV^{-1} (the vector representation) if $\vec{\sigma}.\vec{\mathbf{x}}$ transforms into $V\ \vec{\sigma}.\vec{\mathbf{x}}\ V^{-1}$ (the vector linear representation). The vector $\vec{\mathbf{v}}$ obtained from the equation $V \ = \ exp\ (i/2\ \ \vec{\sigma}.\vec{\mathbf{v}})$ defines the rotation axis and angle in correspondence with $SO(3)$ rotations. $\vec{\mathbf{x}}$ provides the space coordinates and transforms like a $SO(3)$ real vector, but being a $SU(2)$ parameter it varies on a spherical volume of radius $2\pi t_0$, $x \ \leq \ 2\pi t_0$, whose surface is identified to a single point. Under the rotation defined by V , $\vec{\mathbf{x}}$ transforms into $R_V\vec{\mathbf{x}} \ = \ \vec{\mathbf{x}}_L \ + \ cos\ v\ \vec{\mathbf{x}}_\perp \ + \ v^{-1}\ sin\ v\ \vec{\mathbf{x}} \ \wedge \ \vec{\mathbf{v}}$, where $\vec{\mathbf{x}}_L \ = \ v^{-2}\ (\vec{\mathbf{x}}.\vec{\mathbf{v}})\ \vec{\mathbf{v}}$ and $\vec{\mathbf{x}_\perp} \ = \ \vec{\mathbf{x}} - \vec{\mathbf{x}}_L$, as it should be the case for a genuine space rotation. The observer can thus feel a vector space in a spinorial space-time.

The cosmic time scale given by the radius of the Universe does not correspond to the local time scale of physical processes at time t , which depends on the local vacuum dynamics and varies with cosmic time. In situations where the Universe evolves smoothly, we can identify both time scales up to a constant to be determined locally. Whatever the relation between the two time scales, the above example (which requires by itself unification of space and time) clearly shows that abandoning textbook relativity dogmas does not necessarily destroy the standard cosmological framework.

3. SPACE-TIME SYMMETRIES AND WAVE FUNCTIONS

On a three-dimensional vector representation, a 2π rotation is equal to the identity, contrary to spinor representations of $SU(2)$ where the same transformation changes the sign of the spinors. The description of particles with half-integer spin in a real space-time has always posed some conceptual problems, in the sense that a spinor wave function cannot be built by standard operations from representations of the space rotation group. More precisely, a particle with half-integer spin cannot be described by a single-valued function of the standard space and time variables. To circumvent this difficulty, it is said that spinors do not form representations of the basic symmetry group $SO(3)$ of space but of the covering group $SU(2)$ of its Lie algebra, or that they belong to representations of the rotation group "up to a sign". The same situation arises in a relativistic description, where $SL(2\ ,\ C)$ is the covering group of the Lie algebra of the Lorentz group which, when complexified, is equivalent to that of $SL(2\ ,\ C)_{left}\ \otimes\ SL(2\ ,C)_{right}$ (for "left" and "right" chiral spaces). Our spinorial space-time naturally provides a more compact description: position in the spinor space-time is described by a spinor whereas, simultaneously and independently of whether relativity applies, position in the $\mid \xi \mid \ = \ t_0$ hypersphere (the "space" at time $t = t_0$) is described by a vector. Spinor fields become then single-valued functions of space-time coordinates. It seems worth emphasizing again that, even without relativity, our approach requires and operates a geometric unification of space and time.

At fixed t_0 , the position vector $\vec{\mathbf{x}}$ describes the relative position of the point ξ with respect to an observer placed at ξ_0 . The space coordinates are the parameters of the (unique) element U of $SU(2)$ which transforms ξ_0 into ξ . Contrary to the conventional description of space rotations, no $SU(2)$ transformation other than the identity leaves the observer position unchanged, but the observer can measure only the relative position of surrounding objects. A 2π rotation would change the signs of ξ_0 and ξ simultaneously.

It forms with the identity the center of the group and cannot be felt by any position measurement (the observer is by definition insensitive to its own motion), although the properties of spin-1/2 particles can be checked by other experiments. Therefore, in our approach, a space rotation around an axis passing through the observer position is a $SU(2)$ transformation whose effects cannot be entirely seen through position measurements. If V is the $SU(2)$ transformation, all the points in the spinor space of the form $\xi = V^\lambda \xi_0$, where λ is a real number between $-2\pi v^{-1}$ and $2\pi v^{-1}$, appear unchanged to the observer when V acts on the spinor space-time. However, V actually changes all these points (they move along the rotation axis) and preserves only the overall geodesic on the $\xi = t_0$ hypersphere. Space rotations significantly transform the whole Universe, and can deplace the observer at cosmic scales. While space rotations correspond to $SU(2)$ transformations, space translations correspond to a change in the position of the observer on the $\xi = t_0$ hypersphere. Moving the observer from ξ_0 to $\xi'_0 = W\xi_0$ changes the above defined U into UW^{-1} and, writing $W = exp\ (-i/2\ \ t_0^{-1}\ \vec{\sigma}.\vec{\mathbf{w}})$, $\vec{\sigma}.\vec{\mathbf{x}}$ is changed into $-2it_0\ ln\ (UW^{-1})$, where ln stands for neperian logarithm uniquely defined, for $U \neq -W$, in the range of traceless hermitic 2×2 matrices with eigenvalues $\pm\lambda$ such that $\mid \lambda \mid \ < 2\pi t_0$. In the "local" limit $x/t_0 \ll 1$ and $w/t_0 \ll 1$, the expression $-2it_0\ ln\ (UW^{-1})$ can be approximated by $\vec{\sigma}.(\vec{\mathbf{x}} + \vec{\mathbf{w}})$. Space translations commute in the infinitesimal limit where the structure of the Lie algebra (providing the cosmic curvature) can be ignored. The usual parametrization of space transformations is a local one, based on our intuitive, infinitesimal view of the tangent hyperplane to the $\xi = t_0$ hypersphere at the observer position.

Contrary to space translations, space-time translations defined using spinorial coordinates are real translations which commute but which cannot leave invariant any $\mid \xi \mid =$ constant hypersphere. We expect them to make sense locally, at scales much smaller than the size of the Universe. The generators of such translations are $SU(2)$ spinors, and so is the space-time position spinor $\Delta\xi = \xi - \xi_0$ of any point of space-time with respect to an observer placed at ξ_0 . Supersymmetry could possibly be connected to derivation with respect to the spinorial coordinates, but we shall not discuss this point in the present work.

Because of the expansion of the space hypersphere, we do not really expect invariance under time translations (energy conservation) to hold at cosmic scale, even if it can be a good local symmetry in the infinitesimal limit $\Delta t \ll t$ where our intuition suggests energy conservation to be a basic law of Nature. Similar considerations apply to discrete time symmetries, such as T and CPT (no longer protected by a universal Lorentz invariance), as the two directions of time are clearly inequivalent at cosmic scale. As will be seen later, we expect the cosmological redshift and time dilation to apply as in standard cosmology.

Up to a n-dependent normalization factor, and taking the above $d^3\vec{\mathbf{x}}^*$ as the volume element, an example of spinor wave function at time t could be, for integer $n \geq 0$:

$$\psi_n(\xi) = t^{-3/2}\ (\psi_0^\dagger\ U\ \psi_0)^n\ \psi_0 \tag{6}$$

and, for integer $n \leq 0$:

$$\psi_{-n}\ (\xi) = t^{-3/2}\ (\psi_0^\dagger\ U^\dagger\ \psi_0)^n\ \psi_0 \tag{7}$$

where ξ_0 is the observer position at time t , $\psi_0 \ = \ \psi \ (\xi_0)$ is the value of the spinor wave function at ξ_0 and U is the operator uniquely defined by equation (1) , tranforming ξ_0 into ξ . The quantization of n is required by the regularity of the wave function at zero values of the matrix elements $\psi_0^\dagger \ U \ \psi_0$. In realistic quantum-mechanical situations, n is a very large number in order to make the wave function oscillate at small wavelengths. The function defined by the n-th power of the $n \ = \ 1$ wave function $\psi_0^\dagger \ U \ \psi_0$, or by its complex conjugate, is then close to a plane wave, as can be seen in what follows.

From the parametrization (1) , we can replace ξ by its space coordinate vector $\vec{\mathbf{x}}$ and write for a spinor wave function $\psi \ = \ \psi \ (\vec{\mathbf{x}} \ , \ t)$. Let us the consider the case where ψ_0 is an eigenspinor of $\vec{\sigma}.\vec{\mathbf{x}}$, and set a coordinate system where $\vec{\mathbf{x}} = (x \ , \ 0 \ , \ 0)$. Then, if σ_1 is taken to be diagonal and $\sigma_1 \ \psi_0 \ = \ \psi_0$, one has $\psi_0^\dagger \ U \ \psi_0 \ = \ exp \ (it^{-1}x/2)$. If, instead, $\vec{\mathbf{x}} \ = \ (x_1 \ , \ x_2 \ , \ x_3)$, and if $x_2 \ , x_3 \ \ll \ t$, one has $\psi_0^\dagger \ U \ \psi_0 \ \simeq \ exp \ (it^{-1}x_1/2)$. In both cases, the wave function can be approximated by a plane wave with wave vector $(t^{-1}/2 \ , \ 0 \ , \ 0)$. Similarly, the n-th power of the matrix element will be a plane wave with momentum $(nt^{-1}/2 \ , \ 0 \ , \ 0)$ and can be approximated by $exp \ (i \ n \ t^{-1} \ x_1/2)$ as long as $n \ (x^2 \ - \ x_1^2) \ \ll \ t^2$. The above considered translation W , acting on the spinor wave functions, will turn U into UW^{-1} . It will, to leading order, transform $\vec{\mathbf{x}}$ into $\vec{\mathbf{x}} \ + \ \vec{\mathbf{w}}$ and the wave function $\psi_n \ \propto \ exp \ (i \ n \ t^{-1} \ x_1/2)$ into $\psi'_n \ \propto \ exp \ [i \ n \ t^{-1} \ (x_1 \ + \ w_1)/2]$ as long as $n \ (w^2 \ - \ w_1^2) \ \ll \ t^2$. On an arbitrary spinor wave function ψ_n (and similarly for ψ_{-n} via complex conjugation), of the form (6) , the generators of space translations are the matrices $in/2 \ \sigma_j \ t^{-1} \ (j \ = \ 1 \ , \ 2 \ , \ 3)$ multiplying U at right inside the matrix element $\psi_0^\dagger U \psi_0$. Taking $t \ \approx \ 10^{26} \ m$ for cosmic time, the above wave function can be approximated by a plane wave for $x_\perp^2 \ = \ x^2 \ - \ x_1^2 \ \lesssim \ 10^{26} \ m \ k_L^{-1}$ where $k_L \ = \ nt^{-1}/2$ is the longitudinal wave vector scale. If $n \ \approx 10^{45}$ and $k_L^{-1} \ \approx 10^{-19} \ m$ (the lowest distance scale accessible to experiments at TeV energies), the plane wave approximation holds for $| \ x_\perp \ | \ \lesssim \ 1 \ Km$ (gaussian damping). If $n \ \approx 10^{61}$ and $k_L^{-1} \ \approx 10^{-35} \ m$, the approximation is valid for $| \ x_\perp \ | \ \lesssim \ 10^{-5} \ m$. Assuming that this transverse damping of quantum wave functions at high wavelength is to be taken seriously, it is not clear whether feasible experiments can eventually measure such an effect related to the possible finite radius of the Universe.

If ψ_0 is an eigenspinor of σ_1 with eigenvalue s (necessarily 1 or -1), and with the same approximations as before, ψ_n will be a momentum eigenstate with wave vector $\vec{\mathbf{k}}$ of components $(nst^{-1}/2 \ , \ 0 \ , \ 0)$. It will therefore, with the same approximations, satisfy the equation:

$$\vec{\sigma}.\vec{\mathbf{k}} \ \ | \ \psi_n > \ \ = \ \ s \ k \ n \ \ | \ n \ |^{-1} \ \ | \ \psi > \ \ = \ \ \lambda \ k \ \ | \ \psi_n > \qquad (8)$$

where $\lambda \ = \ s \ n \ | \ n \ |^{-1}$ is the helicity, with n taking integer values from (6) and (7).

4. EQUATIONS OF MOTION

In the present geometric approach, a natural cosmic time is given by the size of the Universe. If radial directions from the point $\xi = 0$ define the arrow of time and the motion of an absolute local rest frame (the "vacuum rest frame"), it appears from continuity and from quantization due to the compactness of three-dimensional space, e.g. in (6) , that the wave vector $\vec{\mathbf{k}}$ of a quantum particle must evolve in time like t^{-1} , generating a geometric shift of wavelengths due to the expansion of the Universe. This is not surprising, as it just corresponds to the geometric version of the standard cosmological redshift. Since our cosmic time t is identical to the radius of the Universe, we immediately recover the well-known expression (e.g. [7]):

$$\lambda_o \lambda_e^{-1} \quad = \quad a_o a_e^{-1} \tag{9}$$

where λ_o is the observed wavelength of the particle at the time of its detection, a_o the expansion parameter of the Universe at detection time, λ_e the emission wavelength and a_e the value of the expansion parameter at the time of emission of the particle. The recession rate, given by Hubble's constant, is very small as compared to laboratory time scales. In the rest of the chapter, we will restrict ourselves to local phenomena and ignore effects related: a) to time evolution at cosmic scale; b) to the curvature of space at large scales. In a global description, the speed $\vec{\mathbf{v}} = d\vec{\mathbf{x}}/dt$ should be replaced by $\vec{\mathbf{v}^*} = \vec{\mathbf{dx}}^*/dt$, and the gradient with respect to $\vec{\mathbf{dx}}$ by a gradient with respect to $\vec{\mathbf{dx}}^*$. Unless otherwise stated, we consider in what follows the limit where the Universe is very large as compared to local space and time scales for laboratory phenomena, and therefore we approximate the wave vector spectrum by a continuum spectrum.

4a. Classical mechanics and kinematics

To illustrate classical mechanics with superluminal particles, assume a system of N interacting particles in the vacuum rest frame with: a) different critical speeds c_1 , c_2 , ... , c_N associated to different sectorial Lorentz invariances; b) the following lagrangian in the local rest frame of vacuum:

$$L \quad = \quad - \ \Sigma_{i=1}^N \ m_i \ c_i^2 \ (1 \ - \ v_i^2 c_i^{-2})^{1/2} \quad - \quad U(\vec{\mathbf{x_1}} \ , \ \vec{\mathbf{x_2}} \ , \ ... \ , \ \vec{\mathbf{x_N}}) \tag{10}$$

where: a) v_i is the modulus of $\vec{\mathbf{v_i}}$; b) the vectors $\vec{\mathbf{v_i}}$ and $\vec{\mathbf{x_i}}$ stand for the speed and position of particle i ; c) m_i is the inertial mass of particle i ; d) U is a potential energy describing the interaction, where we ignore large scale geometric effects related to the space coordinates. This lagrangian implies that all sectorial Lorentz invariances (i.e. all sectorial Lorentz metrics in their canonical diagonal form) can be simultaneously exhibited in a single rest frame, the "absolute" or "vacuum" rest frame. More complicate scenarios can be imagined. Standard use of the variational principle leads from (10) to the equations of motion ($i \ = \ 1 \ , \ 2 \ , \ ... \ , \ N$):

$$d/dt \ \vec{\mathbf{p_i}} \quad = \quad - \ \vec{\nabla}_{\mathbf{i}} \ U \tag{11}$$

where $\vec{\nabla}_{\mathbf{i}}$ means partial gradient with respect to the position of the i-th particle and:

$$\vec{\mathbf{p_i}} = m \ \vec{\mathbf{v_i}} \ (1 \ - \ v_i^2 c_i^{-2})^{-1/2} \tag{12}$$

As in standard mechanics, momentum conservation holds:

$$d/dt \ (\Sigma_{i=1}^N \ \vec{\mathbf{p_i}}) \quad = \quad 0 \tag{13}$$

and energy conservation is expressed by the formula:

$$d/dt\ (\Sigma_{i=1}^{N}\ \vec{\mathbf{p_i}}.\vec{\mathbf{v_i}}\ \ -\ \ L\ \ ) = \ \ d/dt\ (\Sigma_{i=1}^{N}\ p_i^0 c_i\ \ +\ \ U)\ \ =\ \ 0 \tag{14}$$

where:

$$p_i^0\ \ =\ \ E_i c_i^{-1}\ \ =\ \ m_i c_i\ (1\ -\ v_i^2 c_i^{-2})^{-1/2} \tag{15}$$

and E_i is the non-interacting (i.e. rest + kinetic) energy of particle i . $\mathcal{E}\ = \ \Sigma_{i=1}^{N}\ p_i^0 c_i\ +\ U$ is the total energy of the system. The 4-vectors $(p_i^0\ ,\ \vec{\mathbf{p_i}})$ transform covariantly under Lorentz tranformations with critical speed parameter c_i . However, the conservation laws involving several sectors with different values of the c_i's are not covariant and therefore can be written in the above form only in the vacuum rest frame. "Lorentz" transformations to other inertial frames will depend on the matter the observer is made of. Since we expect to measure the energy of superluminal particles through interactions with "ordinary" particles, we can define, in the rest frame of an "ordinary" particle moving at speed $\vec{\mathbf{V}}$ with respect to the vacuum rest frame, the energy and momentum of a superluminal particle to be the Lorentz-tranformed of its vacuum rest frame energy and momentum taking c as the critical speed parameter for the Lorentz transformation. Then, the mass of the superluminal particle will depend on the inertial frame. The energy and momentum of particle i in the new rest frame, as measured by ordinary matter from energy and momentum conservation (e.g. in decays of superluminal particles into ordinary ones), will be:

$$E'_i\ \ =\ \ (E_i\ -\ \vec{\mathbf{V}}.\vec{\mathbf{p_i}})\ (1\ -\ V^2 c^{-2})^{-1/2} \tag{16}$$

$$\vec{\mathbf{p'_i}}\ \ =\ \ \vec{\mathbf{p'_{i,L}}}\ +\ \vec{\mathbf{p'_{i,\perp}}} \tag{17}$$

$$\vec{\mathbf{p'_{i,L}}}\ \ =\ \ (\vec{\mathbf{p_{i,L}}}\ -\ E_i\ c^{-2}\ \vec{\mathbf{V}})\ (1\ -\ V^2 c^{-2})^{-1/2} \tag{18}$$

$$\vec{\mathbf{p'_{i,\perp}}}\ \ =\ \ \vec{\mathbf{p_{i,\perp}}} \tag{19}$$

where $\vec{\mathbf{p_{i,L}}}\ = \ V^{-2}\ (\vec{\mathbf{V}}.\vec{\mathbf{p_i}})\ \vec{\mathbf{V}}$, $\vec{\mathbf{p_{i,\perp}}}\ = \ \vec{\mathbf{p_i}}\ -\ \vec{\mathbf{p_{i,L}}}$ and similarly for the longitudinal and transverse components of $\vec{\mathbf{p'_i}}$. We are thus led to consider the effective squared mass:

$$M_{i,c}^2\ \ =\ \ c^{-4}\ (E_i^2\ -\ c^2 p_i^2)\ \ =\ \ m_i^2 c^{-4} c_i^4\ +\ c^{-2}(c^{-2}c_i^2\ -\ 1)\ p_i^2 \tag{20}$$

which depends on the vacuum rest frame momentum of the particle. m_i is the invariant mass of particle i , as seen by matter from the i-th superluminal sector (i.e. with critical speed in vacuum = c_i). While "ordinary" transformation laws of energy and momentum are not singular, even for a superluminal particle, the situation is different for the transformation of a superluminal speed, as will be seen below. Furthermore, a mathematical surprise arises: assume $\vec{\mathbf{v_i}}\ =\ \vec{\mathbf{V}}$, so that particle i is at rest in the new inertial rest frame. Then, we would naively expect a vanishing momentum, $\vec{\mathbf{p'_i}}\ =\ \mathbf{0}$. Instead, we get:

$$\vec{\mathbf{p'_i}}\ \ =\ \ -\ \vec{\mathbf{p_i}}\ (c^{-2}c_i^2\ -\ 1)\ (1\ -\ V^2 c^{-2})^{-1/2} \tag{21}$$

and $p'_i\ \gg\ p_i$, although $p'_i c\ \ll\ E'_i$ if $V\ \ll\ c$. This reflects the non-covariant character of the 4-momentum of particle i under "ordinary" Lorentz transformations. Thus, even if the directional effect is small in realistic situations (f.i. on earth), the decay of a superluminal particle at rest into ordinary particles will not lead to an exactly vanishing total momentum if the inertial frame is different from the vacuum rest frame.

In the rest frame of an "ordinary" particle moving with speed $\vec{\mathbf{V}}$ with respect to the vacuum rest frame, we can estimate the speed $\vec{\mathbf{v}}'_{\mathbf{i}}$ of the previous particle i writing:

$$\vec{\mathbf{v}}_{\mathbf{i}} \;=\; \vec{\mathbf{v}}_{\mathbf{i},\mathbf{L}} \;+\; \vec{\mathbf{v}}_{\mathbf{i},\perp} \tag{22}$$

where $\vec{\mathbf{v}}_{\mathbf{i},\mathbf{L}} = V^{-2}(\vec{\mathbf{V}}.\vec{\mathbf{v}}_{\mathbf{i}})\;\vec{\mathbf{V}}$, $\vec{\mathbf{v}}_{\mathbf{i},\perp} = \vec{\mathbf{v}}_{\mathbf{i}} - \vec{\mathbf{v}}_{\mathbf{i},\mathbf{L}}$ and similarly for the longitudinal and transverse components of $\vec{\mathbf{v}}'_{\mathbf{i}}$. Then, the transformation law is:

$$\vec{\mathbf{v}}'_{\mathbf{i},\mathbf{L}} \;=\; (\vec{\mathbf{v}}_{\mathbf{i},\mathbf{L}} \;-\; \vec{\mathbf{V}})\;(1 \;-\; \vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}}\;c^{-2})^{-1} \tag{23}$$

$$\vec{\mathbf{v}}'_{\mathbf{i},\perp} \;=\; \vec{\mathbf{v}}_{\mathbf{i},\perp}\;(1 \;-\; V^2c^{-2})^{1/2}\;(1 \;-\; \vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}}\;c^{-2})^{-1} \tag{24}$$

leading to singularities at $\vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}} \;=\; c^2$ which correspond to a change in the arrow of time (due to the Lorentz transformation of space and time with respect to the "absolute" rest frame) as seen by ordinary matter traveling at speed $\vec{\mathbf{V}}$ with respect to the vacuum rest frame. At $v_{i,L} \;>\; c^2V^{-1}$, a superluminal particle moving forward in time in the vacuum rest frame will appear as moving backward in time to an observer made of ordinary matter and moving at speed $\vec{\mathbf{V}}$ in the same frame. On earth, taking $V \;\approx\; 10^{-3}\;c$ (if the vacuum rest frame is close to that suggested by cosmic background radiation, e.g. [7]), the apparent reversal of the time arrow will occur mainly at $v_i \;\approx\; 10^3\;c$. If $c_i \gg 10^3\;c$, phenomena related to propagation backward in time of produced superluminal particles may be observable in future accelerator experiments slightly above the production threshold. In a typical event where a pair of superluminal particles would be produced, we expect in most cases that one of the superluminal particles propagates forward in time and the other one propagates backward. It must be noticed that, according to (16 - 19), the infinite velocity (value of v'_i) associated to the point of time reversal does not correspond to infinite values of energy and momentum. The backward propagation in time, as observed by devices which are not at rest in the vacuum rest frame, is not really physical (the arrow of time is well defined in the vacuum rest frame for all physical processes) and does not correspond to any real violation of causality (see also Subsection 4d). The apparent reversal of the time arrow for superluminal particles at $\vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}} \;>\; c^2$ would be a consequence of the bias of the laboratory time measurement due to our motion with respect to the absolute rest frame. The distribution and properties of such events, in an accelerator experiment or in a large volume cosmic ray detector, would obviously be in correlation with the direction and speed of the laboratory's motion with respect to the absolute rest frame and provide fundamental cosmological information, complementary to cosmic microwave background.

From (23) and (24), we also notice that, for $V \;\ll\; c$ and $\vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}} \;\gg\; c^2$, the speed $\vec{\mathbf{v}}'_{\mathbf{i}}$ tends to the limit $\vec{\mathbf{v}}_{\mathbf{i}}^{\infty}$, where:

$$\vec{\mathbf{v}}_{\mathbf{i}}^{\infty}\;(\vec{\mathbf{v}}_{\mathbf{i}}) \;=\; -\;\vec{\mathbf{v}}_{\mathbf{i}}\;c^2\;(\vec{\mathbf{v}}_{\mathbf{i}}.\vec{\mathbf{V}})^{-1} \tag{25}$$

which sets a universal high-energy limit, independent of c_i , to the speed of superluminal particles as measured by ordinary matter in an inertial rest frame other than the vacuum rest frame. This limit is not isotropic, and depends on the angle between the speeds $\vec{\mathbf{v_i}}$ and $\vec{\mathbf{V}}$. A typical order of magnitude for $\vec{\mathbf{v_i}}^{\infty}$ on earth is $\vec{\mathbf{v_i}}^{\infty} \approx 10^3 \, c$ if the vacuum rest frame is close to that suggested by cosmic background radiation. If C is the highest critical speed in vacuum, infinite speed and reversal of the arrow of time occur only in frames moving with respect to the vacuum rest frame at speed $V \geq c^2 C^{-1}$. Finite critical speeds of superluminal sectors, as measured by ordinary matter in frames moving at $V \neq 0$, are anisotropic. Therefore, directional detection of superluminal particles would allow to directly identify the effective vacuum rest frame for each superluminal sector.

4b. Non-covariance of dynamics

Interaction between particles from different dynamical sectors is, for simple physical reasons, expected to depend on the rest frame of the system. For instance, Lorentz contraction has an intrinsic physical meaning in the vacuum rest frame and, at equal speed, is different for particles from different dynamical sectors. Therefore, the relative size of two such particles moving with the same speed will depend on their motion with respect to the absolute rest frame. This should in principle influence their interaction properties.

Assuming that the above particles can be dealt with as spherical extended objects of radius r_i , and neglecting spin, we can as an example attribute to particle i , with position $\vec{\mathbf{x_i}}$ and moving at speed $\vec{\mathbf{v_i}}$, a form factor depending on $\vec{\mathbf{x}} - \vec{\mathbf{x_i}}$:

$$F_i \, (\vec{\mathbf{x}} \, , \, \vec{\mathbf{x_i}} \, , t) \; = \; f_i \, [(\vec{\mathbf{x_L}} \, - \, \vec{\mathbf{x}}_{\mathbf{i,L}} \, - \, \vec{\mathbf{v_i}} \, t)^2 \, r_i^{-2} \, (1 \, - \, v_i^2 c_i^{-2})^{-1} \; + \; (\vec{\mathbf{x}}_\perp \, - \, \vec{\mathbf{x}}_{\mathbf{i},\perp})^2 \, r_i^{-2}] \quad (26)$$

where $\vec{\mathbf{x}}_{\mathbf{i,L}} = v_i^{-2}(\vec{\mathbf{v_i}}.\vec{\mathbf{x_i}}) \; \vec{\mathbf{v_i}}$, $\vec{\mathbf{x}}_{\mathbf{i},\perp} = \vec{\mathbf{x_i}} - \vec{\mathbf{x}}_{\mathbf{i,L}}$ and similarly for the longitudinal and transverse components of $\vec{\mathbf{x}}$ with respect to the direction of $\vec{\mathbf{v_i}}$. Taking as before $\vec{\mathbf{V}} = \vec{\mathbf{v_i}}$, we find under an "ordinary" Lorentz transformation with relative speed $\vec{\mathbf{V}}$ the transformed form factor $F'_i \, (\vec{\mathbf{x'}} \, , \, \vec{\mathbf{x'}} \, , \, t')$ given by:

$$F'_i \; = \; f_i \, [(\vec{\mathbf{x'_L}} \, - \, \vec{\mathbf{x}}'_{\mathbf{i,L}})^2 \, r_i^{-2} \, (1 \, - \, V^2 c_i^{-2})^{-1} \, (1 \, - \, V^2 c^{-2}) \; + \; (\vec{\mathbf{x'}}_\perp \, - \, \vec{\mathbf{x}}'_{\mathbf{i},\perp})^2 \, r_i^{-2}] \quad (27)$$

Where $\vec{\mathbf{x'}}$ and $\vec{\mathbf{x'_i}}$ are the Lorentz-transformed coordinates and their longitudinal and transverse components with respect to the direction of $\vec{\mathbf{V}}$ are defined in the same way as before. The relative longitudinal size $l_i \, l_0^{-1}$, where l_0 and l_i are respectively the length of an ordinary particle and of particle i taken to be at rest in the vacuum rest frame, turns under the above Lorentz transformation into $l_i \, l_0^{-1} \, (1 \, - \, V^2 c^{-2})^{-1/2} \, (1 \, - \, V^2 c_i^{-2})^{1/2}$ in the new intertial frame if, in both frames, the length is measured in the direction of $\vec{\mathbf{V}}$. When $V \rightarrow c$, the longitudinal size of a superluminal particle, as measured by ordinary matter, will tend to infinity: the ordinary particles become infinitely thin in the longitudinal direction, as compared to the superluminal ones. Dynamics should be sensitive to this contraction, which reflects the interaction of moving particles with the vacuum.

To obtain a quantum lagrangian corresponding to the classical lagrangian (9), we can write $\vec{\mathbf{p_i}} = (h/2\pi) \, \vec{\mathbf{k_i}}$, where h is the Planck constant, and:

$$\vec{\mathbf{v_i}} \; = \; \vec{\mathbf{p_i}} \, c_i \, (p_i^2 \, + \, m_i^2 c_i^2)^{-1/2} \quad (28)$$

from which, assuming for simplicity the particles to be spinless and neutral, we can according to (10) build for particle i the free lagrangian:

$$L_{i,free} \quad = \quad - \int (k_i^0)^{-1} \ d^3\vec{\mathbf{k_i}} \ \mathbf{a_i^\dagger}(\vec{\mathbf{k_i}})\mathbf{a_i}(\vec{\mathbf{k_i}}) \ m_i^2 c_i^3 \ [(2\pi)^{-2} h^2 k_i \ ^2 + \ m_i^2 c_i^2]^{-1/2} \tag{29}$$

where $k_i^0 \ = \ [k_i \ ^2 + \ 4\pi^2 \ h^{-2} \ m_i^2 c_i^2]^{-1/2} \ = \ 2\pi \ p_i^0 h^{-1}$, $\mathbf{a_i^\dagger}(\vec{\mathbf{k_i}})$ creates a particle of type i with wave vector $\vec{\mathbf{k_i}}$, and $\mathbf{a}(\vec{\mathbf{k_i}})$ annihilates such a particle. The normalization constraint for the operators is:

$$N_i \quad = \quad \int (k_i^0)^{-1} \ d^3\vec{\mathbf{k_i}} \ \mathbf{a_i^\dagger}(\vec{\mathbf{k_i}})\mathbf{a_i}(\vec{\mathbf{k_i}}) \tag{30}$$

where N_i is the total number of particles of type i. Assuming the above particles to be scalars, and following the standard construction of free quantum fields (e.g. [8]), the creation and annihilation operators can be written in terms of the the scalar field $\Phi_i(\vec{\mathbf{x}} \ , \ t)$:

$$\mathbf{a_i}^\dagger \ (\vec{\mathbf{k_i}}) \quad = \quad (8\pi)^{-3/2} \ \ [\alpha_\mathbf{i}^\dagger \ (\vec{\mathbf{k_i}}) \ + \ \beta_\mathbf{i}^\dagger \ (\vec{\mathbf{k_i}})] \tag{31}$$

where:

$$\alpha_\mathbf{i}^\dagger \ (\vec{\mathbf{k_i}}) \quad = \quad k_i^0 \int d^3\vec{\mathbf{x}} \ \ exp \ [i \ (\vec{\mathbf{k_i}}.\vec{\mathbf{x}} \ - \ k_i^0 c_i t)] \ \ \Phi_i(\vec{\mathbf{x}} \ , \ t) \tag{32}$$

$$\beta_\mathbf{i}^\dagger \ (\vec{\mathbf{k_i}}) \quad = \quad - \ ic_i^{-1} \int d^3\vec{\mathbf{x}} \ \ exp \ [i \ (\vec{\mathbf{k_i}}.\vec{\mathbf{x}} \ - \ k_i^0 c_i t)] \ \ \partial/\partial t \ \Phi_i(\vec{\mathbf{x}} \ , \ t) \tag{33}$$

and similarly for $\mathbf{a_i}(\vec{\mathbf{k_i}})$. We are thus led to the free lagrangian:

$$L_{i,free} \quad = \quad \int d^3\vec{\mathbf{x}} \ \mathcal{L}_i \tag{34}$$

where:

$$\mathcal{L}_i \quad = \quad - \ (4\pi)^{-1} \ h \ c_i \ [m_i^2 c_i^2 \ (h/2\pi)^{-2} \ \Phi_i^2 \ \ - \ c_i^{-2} \ (\partial\Phi_i/\partial t)^2 \ + \ (\vec{\nabla}\Phi_i)^2] \tag{35}$$

and we can associate to (10) the quantum lagrangian:

$$L \quad = \quad \Sigma_{i=1}^N \ \ L_{i,free} \ \ + \ \ L_{int} \tag{36}$$

where L_{int} is the interaction lagrangian. Each sectorial free lagrangian density $\mathcal{L}_i$ is a scalar with respect to Lorentz transformations with c_i as the critical speed parameter, but not with respect to other Lorentz transformations. In spite of the loss of universal Lorentz covariance, there seems to be no obvious effect tending to spoil the consistency of the quantum field theory involving superluminal particles (see also Subsection 4d).

4c. Quantum wave equations

In the previous subsection, we considered the situation where, in the vacuum rest frame, all free quantum particles satisfy Klein-Gordon equations with c_i as the critical speed parameter for particle i . This was inspired by our knowledge of "ordinary" particles. But we can also address the following question: what is the most general local wave equation for a particle in the Universe we just described? Assuming that the $SU(2)$ invariant equations of motion are linear in the wave function ϕ and can be formulated locally in the absolute local rest frame in terms of first and second-order derivatives, a free particle wave function at time $t = t_0$ will satisfy in the vacuum rest frame the equation:

$$- A \, \partial^2 \phi / \partial t^2 + iB \partial \phi / \partial t + \Sigma_{j=1}^3 \partial^2 \phi / \partial x_j^2 - D \, \phi = 0 \tag{37}$$

where A , B and D are functions of t and the x_j are the real coordinates of the position vector $\vec{\mathbf{x}}$. As long as effects at cosmic scale can be ignored, we can to a first approximation assume that A , B and D have constant values and neglect the large scale time evolution of frequencies and wave vectors due to the cosmological redshift. In such a scenario, if B and D can be neglected, the cosmological redshift implies as in [7] that the emitted and observed frequencies ν_e and ν_o of radiation will be related by the equation:

$$\nu_o \, a_o = \nu_e \, a_e \tag{38}$$

which also applies to proper rates of events (time dilation). Thus, our scenario is close to the natural predictions of standard cosmology. If B and D do not vanish (they give the mass and rest energy of the particle), the dependence on cosmic time of BA^{-1} and DA^{-1} may be a nontrivial problem. A can be set constant by local time rescaling, which implicitly modifies the local time scale to describe the evolution of the Universe.

In the case (forbidden by Lorentz invariance) where $B \neq 0$, equation (37) is not self-conjugate. Assuming, for simplicity, that ϕ is a scalar, its complex conjugate ϕ^* will satisfy in the vacuum rest frame the wave equation:

$$- A \, \partial^2 \phi^* / \partial t^2 - iB \partial \phi^* / \partial t + \Sigma_{j=1}^3 \partial^2 \phi^* / \partial x_j^2 - D \, \phi^* = 0 \tag{39}$$

In terms of energy and momentum, the solutions of equation (37) are:

$$E = (4\pi)^{-1} h \, A^{-1} \, [B + (B^2 + 4Ak^2 + 4AD)^{1/2}] \qquad (solution \ 1) \tag{40}$$

$$E = (4\pi)^{-1} h \, A^{-1} \, [B - (B^2 + 4Ak^2 + 4AD)^{1/2}] \qquad (solution \ 2) \tag{41}$$

whereas equation (39) admits the solutions:

$$E = (4\pi)^{-1} h \, A^{-1} \, [-B + (B^2 + 4Ak^2 + 4AD)^{1/2}] \qquad (solution \ 3) \tag{42}$$

$$E = (4\pi)^{-1} h \, A^{-1} \, [-B - (B^2 + 4Ak^2 + 4AD)^{1/2}] \qquad (solution \ 4) \tag{43}$$

where, as usual, $\vec{\mathbf{p}} = h \, (2\pi)^{-1} \, \vec{\mathbf{k}}$ and $\vec{\mathbf{k}}$ is the wave vector. The speed of the particle is:

$$\vec{\mathbf{v}} = \vec{\nabla}_{\vec{\mathbf{p}}} \, E = \vec{\mathbf{p}} \, \mathcal{C}^2 \, E_v^{-1} \tag{44}$$

where $E_v = \mathcal{C} \, (p^2 + m^2 \mathcal{C}^2)^{1/2}$, $\mathcal{C} = A^{-1/2}$, $m = (4\pi)^{-1} \, h \, (B^2 + 4D\mathcal{C}^{-2})^{1/2}$ and, solving the equations in $\vec{\mathbf{p}}$, we get the standard relativitic expression:

$$\vec{\mathbf{p}} = m \, \vec{\mathbf{v}} \, (1 - v^2 \mathcal{C}^{-2})^{-1/2} \tag{45}$$

Then, with respect to Lorentz transformations with critical speed parameter $\mathcal{C}$, the energy is a linear combination of: a) a Lorentz scalar, given by the term $E_s = \pm$

$(4\pi)^{-1}\ h\ BA^{-1}$ in (40 - 43); b) the time component of a four-vector, given in (40 - 43) by the term $E_v\ =\ \pm\ (4\pi)^{-1}\ h\ A^{-1}\ (B^2\ +\ 4Ak^2\ +\ 4AD)^{1/2}$. The rest energy and the inertial mass times the square of the critical speed, which are identical in the case of standard relativity, are different in the present case. Assuming $D\ >\ 0$, and following standard procedures of field theory (e.g. [8]), we may consider associating to solutions 1 - 4 in (40 - 43) two scalar fields describing neutral particles: a field Φ_1 for solutions (1) and (4) and a field Φ_2 for solutions (2) and (3). But none of such fields would satisfy a second-order wave equation like (37) or (39). Instead, we may attempt to build a field Φ associated to solutions (1) and (2) and satisfying equation (37). Its conjugate $\Phi^\dagger$ would then correspond to solutions (3) and (4) and satisfy equation (39). In expressions of the same type as (30) and (32), the volume element $(k^0)^{-1}\ d^3\vec{\mathbf{k}}$ should use instead of k^0 its four-vector component $k^0_v\ =\ (k^2\ +\ m^2\mathcal{C}^2)^{1/2}$ ignoring the scalar term $k^0_s\ =\ \pm BA^{-1}/2$, whereas the exponential $exp\ [i\ (\vec{\mathbf{k}}.\vec{\mathbf{x}} - k^0\mathcal{C}t)]$ should use for k^0 the expression $k^0\ =\ k^0_s + k^0_v$. This seems to be the right choice of quantum fields. In this case: a) there would be no symmetry between positive and negative energy solutions inside each field; b) particle and antiparticle would have the same inertial mass, but not the same rest energy. But it would be possible to derive equations (37) and (39) from a free lagrangian density $\mathcal{L}$ in terms of Φ and $\Phi^\dagger$ writing:

$$\mathcal{L}\ =\ -\ (4\pi)^{-1}\ h\ \mathcal{C}\ (\mathcal{L}_0\ +\ i\rho\ \mathcal{L}_\rho) \tag{46}$$

where:

$$\mathcal{L}_0\ =\ m^2\mathcal{C}^2\ (h/2\pi)^{-2}\ \Phi^\dagger\ \Phi\ -\ \mathcal{C}^{-2}\ \partial\Phi^\dagger/\partial t\ \partial\Phi\partial t\ +\ \vec{\nabla}\Phi^\dagger.\vec{\nabla}\Phi \tag{47}$$

$$\mathcal{L}_\rho\ =\ \Phi^\dagger\ \partial\Phi/\partial t\ -\ \partial\Phi^\dagger/\partial t\ \Phi \tag{48}$$

$\rho\ =\ B/2$ and $\mathcal{L}_\rho$ is basically a charge operator accounting, in the lagrangian, for the difference in rest energy between the particle described by Φ and its antiparticle. Obviously, a term proportional to a charge in the lagrangian indicates the existence of a constant potential (a scalar with respect to space rotations, like the electric potential) in the vacuum rest frame. This in turn indicates that an effective charge has condensed in vacuum, and that this charge has locally the same motion as the vacuum rest frame.

Similar considerations apply to fermions. Going back to Section 3 , we can assume that the spinor wave function $\mid\psi_n>$ is locally approximated by a plane wave and, neglecting the cosmological redshift, satisfies in the vacuum rest frame the equation:

$$d/dt\ \mid\psi_n>\ =\ -\,i\ e_n\ \mid\psi_n> \tag{49}$$

which has solution:

$$\psi_n\ (\vec{\mathbf{x}}\ ,\ t)\ =\ exp\ [-ie_n(t-t_0)]\ \psi_n\ (\vec{\mathbf{x}}\ ,\ t_0)\ =\ exp\ [-ie_n(t-t_0)\ +\ i\vec{\mathbf{k}}.\vec{\mathbf{x}}]\ \psi_0 \tag{50}$$

with group velocity $\vec{\mathbf{v}}\ \simeq\ 4\pi\ h^{-1}\ s\ (e_n - e_{n-1})\ t\ \vec{\mathbf{k}}/k$. If the n-dependence of $e_n\ n^{-1}$ can be neglected (massless case), the constant $2\pi h^{-1}\ e_n k^{-1}$ is the critical speed of the particle in vacuum and the helicity is equivalent to chirality. If we apply equations (37) and (39) to the spinor wave function, we get similar solutions for energy in terms of momentum. But we can also linearize the equations introducing the real constants α , δ and ϵ , and writing for (37):

$$(\alpha k_0\ +\ \vec{\sigma}.\vec{\mathbf{k}}\ +\ \delta)\ \psi_1\ \ +\ \ \epsilon\ \psi_2\ =\ 0 \tag{51}$$

$$(\alpha k_0 \ - \ \vec{\sigma}.\vec{\mathbf{k}} \ + \ \delta) \ \psi_2 \ \ + \ \ \epsilon \ \psi_1 \ \ = \ \ 0 \tag{52}$$

which lead to:

$$[(\alpha k_0 \ + \ \delta)^2 \ - \ k^2 \ - \ \epsilon^2] \ \psi_1 \ \ = \ \ [(\alpha k_0 \ + \ \delta)^2 \ - \ k^2 \ - \ \epsilon^2] \ \psi_2 \ \ = \ \ 0 \tag{53}$$

and, for (39):

$$(\alpha k_0 \ + \ \vec{\sigma}.\vec{\mathbf{k}} \ - \ \delta) \ \psi_1^* \ \ + \ \ \epsilon \ \psi_2^* \ \ = \ \ 0 \tag{54}$$

$$(\alpha k_0 \ - \ \vec{\sigma}.\vec{\mathbf{k}} \ - \ \delta) \ \psi_2^* \ \ + \ \ \epsilon \ \psi_1^* \ \ = \ \ 0 \tag{55}$$

leading to:

$$[(\alpha k_0 \ - \ \delta)^2 \ - \ k^2 \ - \ \epsilon^2] \ \psi_1^* \ \ = \ \ [(\alpha k_0 \ - \ \delta)^2 \ - \ k^2 \ - \ \epsilon^2] \ \psi_2^* \ \ = \ \ 0 \tag{56}$$

We are thus led to a pair of complex-conjugate generalized Dirac equations where, in the chiral representation, a constant term has been added to the Dirac operator.

The above discussion points at a possible intrinsic breaking of Lorentz invariance, and of the symmetry between particles and antiparticles, that can be explored experimentally even if the effect is very small. If such a phenomenon happens, and even if it occurs only in a superluminal sector, it can naturally be at the origin of the asymmetry between matter and antimatter in the Universe. A relevant question is how universal would be (if it exists) the value of B inside a given sector. Experiment seems to indicate that A is universal inside the ordinary sector, whereas the effective value of D varies considerably for reasons that may be due to spontaneous symmetry breaking. Lacking a more detailed dynamical description of the phenomenon, we leave this question open.

4d. Causality, spin and statistics

If the number of sectors is finite and all critical speeds are finite, causality is not violated but adapted to the existence of several critical speeds in vacuum. It holds explicitly in the vacuum rest frame, where no signal can propagate faster than the highest critical speed C . If the interaction between sectors with different critical speed in vacuum is weak, we expect by continuity the usual spin-statistics connection to hold, as it holds in the limit where different sectors would not interact. Then, the standard arguments for the spin-statistics connection [9] remain valid with similar assumptions generalized to the new situation. As stressed by many authors (e.g. [10]), causality is the only known principle allowing nowadays to demonstrate the observed relation between spin and statistics.

Even with the breaking of Lorentz invariances due to interaction between different sectors, the existence of a maximum critical speed in the vacuum rest frame and the causality condition with respect to C in this frame, are enough to enforce the standard connexion between spin and statistics for all sectors (the ordinary sector and the superluminal ones). For instance, as discussed in [10], if the canonical quantization for bosons is applied to fermions, the commutator between a free Dirac fermion field at $(t \ , \ \vec{\mathbf{x}})$ and its hermitic conjugate at $(t' \ , \ \vec{\mathbf{x'}})$ is given by a Dirac operator acting on the integral:

$$\Delta_{1,i}(t \ , \ \vec{\mathbf{x}} \ ; \ t' \ , \ \vec{\mathbf{x'}}) \ \ = \ \ (2\pi)^{-3} \int \ (2k_0)^{-1} \ d^3\vec{\mathbf{k}} \ cos[k_0 \ c_i \ (t-t') \ - \ \vec{\mathbf{k}}.(\vec{\mathbf{x}} \ - \ \vec{\mathbf{x'}})] \tag{57}$$

where it can be seen that the function $\Delta_{1,i}(t\ ,\ \vec{\mathbf{x}}\ ;\ t'\ ,\ \vec{\mathbf{x'}})$ and its derivatives do not vanish at $t\ =\ t', \vec{\mathbf{x}}\ \neq\ \vec{\mathbf{x'}}$. Therefore, the incompatibility between causality and the "wrong" statistics remains. Similarly, the compatibility between the "right" statistics and causality survives if causality is understood as being valid in the vacuum rest frame with respect to the highest critical speed C , as common sense suggests.

5. SOME EXPERIMENTAL CONSIDERATIONS

For obvious reasons, the above analysis (which is far from exhausting the basic theoretical problems) was necessary before considering any possible experimental search for superluminal particles. Experimental implications of the present study will be discussed in detail in a forthcoming paper but, from the above discussion, it clearly appears that they are important and that several effects of superluminal sectors may be measurable in future experiments. Contrary to tachyons, which are defined and studied within the original framework of standard relativity [11] considered an absolute property of space-time, the superluminal particles we propose imply by themselves the abandon of this principle (although the space-time felt by our laboratory remains the "ordinary" minkowskian space-time). The new particles have specific experimental signatures, different from those of tachyons [12]. However, since they can travel much faster than light, they can indeed produce some "space-like" phenomena, in astrophysics as well as at accelerators:

- A superluminal particle moving at speed $\vec{\mathbf{v}}$ with respect to the vacuum rest frame, and emitted by an astrophysical object, can reach an observer, moving with laboratory speed $\vec{\mathbf{V}}$ with respect to the same frame, at a time (as measured by the observer) previous to the emission time. Such a phenomenon will happen if $\vec{\mathbf{v}}.\vec{\mathbf{V}}\ >\ c^2$, and the emitted particle will be seen to evolve backward in time (but it evolves forward in time in the vacuum rest frame). If they interact several times with the detector, superluminal particles can be a directional probe preceding the detailed observation of astrophysical phenomena, such as explosions releasing simultaneously neutrinos, photons and superluminal particles. Directional detection of high-speed superluminal particles in a large underground or underwater detector would allow to trigger a dedicated astrophysical observation in the direction of the sky determined by the velocity of the superluminal particle(s). If d is the distance between the observer and the astrophysical object, and Δt the time delay between the detection of the superluminal particle(s) and that of photons and neutrinos, we have: $d\ \simeq\ c\Delta t$.

- Although we expect coupling constants to be small, superluminal particles evolving backward in time can be produced at high-energy accelerators if the energy range $E\ \gtrsim\ m_i\ c_i^2$ is reached for some of such particles. Large detectors with high time resolution may in some cases be able to use timing to identify the production of superluminal particles, especially if they are produced by pairs. If, as suggested by standard cosmology considerations, a typical speed for superluminal particles in our laboratory rest frame is $v\ \approx\ 10^3\ c\ \approx\ 10^{11}\ ms^{-1}$, a detector of radius $\approx 10\ m$ with time resolution $\approx 10^{-9}s$ (not incompatible with the LHC program) would in principle be able to distinguish between ordinary and superluminal particles, at least in some of the relevant events. Larger and faster detectors would be necessary at a later stage in order to measure a high superluminal speed or to identify a particle evolving backward in time, assuming that it interacts sufficiently with the detector (e.g. through "Cherenkov" effect in vacuum).

A neat experimental distinction can be set between the new superluminal particles and the "bradyons" and "tachyons" of ref. [12] . Even if some experimental predictions show analogies between our particles and the tachyons, the main properties of the new particles are closer to those of bradyons (the ordinary subluminal particles of standard particle physics). The new superluminal particles are actually "superbradyons", i.e. bradyons with a critical speed in vacuum higher than c . Like bradyons, they have positive mass as well as positive energy in the laboratory rest frame. Contrary to tachyons [13] , they can emit "Cherenkov" radiation in vacuum [1 - 5] and, although tachyon theory also hints to a vacuum rest frame [14], the physical reason is fundamentally different: there is no need for any "reinterpretation principle" in the case of our superluminal particles.

The analogy with sine-Gordon solitons (see [1 - 5]) can help to understand the difference between our "superbradyons" and the "tachyons" of previous theories. In a galilean space-time, we can build dynamical systems satisfying equations of dalembertian type (i.e. with an invariance identical to Lorentz invariance, but with a different critical speed related to the properties of the dynamical system). Solitons of such systems, if they form a closed self-interacting system, would feel a "relativistic" space-time with their own critical speed playing the role of the speed of light. Such a "micro-universe" would, in some sense, fake our Universe in a finite region of our time. Since the dynamical system has a rest frame, a "vacuum rest frame" is automatically set (like in our Universe, if particles are solitons or topological singularities of vacuum, and if vacuum has a natural rest frame set by cosmology) "Lorentz" contraction in the "micro-universe" is not relative, but absolute, even if a serious but isolated observer made of solitons would be led to formulate a relativity principle and believe erroneously that such a contraction is indeed relative. However, the dalembertian equation is in general a continuum approximation to a discrete system, or the "large distance" limit of a more complex dynamics. It is not valid below a certain distance scale κ^{-1} , where κ is a critical wave vector scale of the system.

The above "micro-universe" is not tachyonic, but bradyonic: solitons have always positive mass and energy. As long as, even with "Lorentz" contraction, the longitudinal size of the soliton remains much larger than κ^{-1} , "Lorentz" invariance guarantees that a soliton, if stable when at rest in the absolute rest frame (the rest frame of the dynamical system), remains stable when it is accelerated to a nonzero speed with respect to the same frame. However, if the soliton is accelerated, with respect to the absolute rest frame, to a speed such that by Lorentz contraction its longitudinal size becomes $\lesssim \kappa^{-1}$, the "relativity principle" felt by the soliton micro-universe does no longer apply. Then, the previously stable soliton can decay into other excitations, like unstable waves, and disperse its energy. A similar discussion can be raised for elementary particles such as quarks and leptons, or their constituents: can we break these particles, just by accelerating them to an inverse distance scale such that relativistic kinematics does no longer hold and the structure of vacuum manifests itself at a deeper level? The existence of very high-energy cosmic rays seems to indicate that such a phenomenon can only happen at energies beyond the reach of present and planned accelerators. However, the subject deserves attention from a long-term point of view, not only for future very high-energy machines but also in the analysis of the highest-energy cosmic rays. In a sectorial grand-unified theory with spontaneous symmetry breaking, it may happen that the estimated mass of a Higgs or intermediate boson be higher than $e_i\ c_i^{-2}\ \approx\ h\ \kappa_i\ c_i^{-1}$, where κ_i is the value of κ for the sector under consideration and e_i is a critical energy scale. It is possible that such a boson will never be formed, and similarly for fermions undergoing the same phenomenon.

The question of whether the constant B (related to some static charge condensed in vacuum) vanishes or not, arises already in the ordinary sector. It could happen that its existence be hidden by inertial masses (typically, if $AD\ \gg\ B^2$). Therefore, it seems sensible to attempt experiments with the so-called "massless" particles (photon and neutrinos) in order to possibly show the effect. If $B\ =\ 0$, a sectorial Lorentz invariance is dynamically generated. A possible evidence for $B\ \neq\ 0$ would be an asymmetry between neutrino and antineutrino oscillations. The cosmological role of the condensed charge, and of the "anomalous" neutrino rest energy, would deserve further investigation.

It turns out from (18), (23) and (24) that, for a high-speed superluminal cosmic ray with critical speed $c_i\ \gg\ c$, the momentum, as measured in the laboratory, does not provide directional information on the source, but on the vacuum rest frame. Velocity provides directional information on the source, but can be measured only if the particle interacts several times with the detector, which is far from guaranteed, or if the superluminal particle is associated to a collective phenomenon emitting also photons or neutrinos simultaneously. ¿From (18), if in the vacuum rest frame the detected particle has speed $v\ \simeq\ c_i$ and energy $E\ \simeq\ p\ c_i$, the momentum $\vec{\mathbf{p}}'$ in the laboratory rest frame will be dominated by the term $-\ E\ c^{-2}\ \vec{\mathbf{V}}\ \simeq\ p\ c_i\ c^{-2}\ \vec{\mathbf{V}}$ if $c_i\ v\ \gg\ c^2$, while the velocity $\vec{\mathbf{v}}'$ is dominated by the term $\vec{\mathbf{v}}\ (1\ -\ \vec{\mathbf{v}}.\vec{\mathbf{V}}\ c^{-2})^{-1}$, and the particle will be seen (in the laboratory) to evolve backward in time if $\vec{\mathbf{v}}.\vec{\mathbf{V}}\ >\ c^2$. Again, standard cosmology suggests that the main speed range for cosmic superluminal particles, as measured on earth by a detector made of ordinary matter, should be $v\ \approx\ 10^3\ c$. This would by itself be a signature. As more schematically emphasized in [5] , one has $E'\ \gg\ p'\ c$, which leads to "back-to-back" events if the superluminal particle transfers most of its energy to a pair of ordinary particles.

In this paper, we have assumed that there exists an absolute rest frame of vacuum, which: a) is the same for all superluminal sectors ; b) provides the local rest frame for the expanding Universe; c) is close to the local rest frame suggested by the study of cosmic background radiation. If superluminal particles are found, it will be possible to directly check the basic hypothesis of this simple scenario. It may happen that the expansion of vacuum has followed different evolutions for the degrees of freedom linked to different sectors or, for instance, that the Universe is not really isotropic because of relative rotational modes between these sets of degrees of freedom (this anisotropy could even be spontaneously generated). In such case, it is likely that a single (local) vacuum rest frame would not exist. The experimental discovery of particles belonging to different superluminal sectors would then (assuming the speed in the laboratory rest frame to be measurable) allow to extract spectra of $\vec{\mathbf{v}}$ and $\vec{\mathbf{p}}$ (see Subsection 4b), perhaps not necessarily in the range $v\ \approx\ 10^3\ c$ or with a different directional dependence, for each dynamical sector. It would then, in any case, be possible for the first time to get a direct insight into the inner structure of vacuum (evidence for a non-empty vacuum has been provided by particle physics, e.g. through the discovery of the $W^\pm$ and Z^0). Ultimately, experiments at very high-energy accelerators or an observatory of superluminal cosmic rays may be able to determine the value of κ_i for each superluminal sector by looking at the energy, momentum and velocity distribution of the events. However, although the basic phenomenon can perhaps be neatly observed, the dynamical interpretation of the data is likely to be far from trivial.

References

[1] L. Gonzalez-Mestres, "Properties of a possible class of particles able to travel faster than light", Proceedings of the Moriond Workshop on "Dark Matter in Cosmology, Clocks and Tests of Fundamental Laws", Villars (Switzerland), January 21-28 1995 , Ed. Frontières, France. Paper astro-ph/9505117 of electronic library.
[2] L. Gonzalez-Mestres, "Cosmological implications of a possible class of particles able to travel faster than light", Proceedings of the Fourth International Workshop on Theoretical and Experimental Aspects of Underground Physics, Toledo (Spain) 17-21 September 1995 , Nuclear Physics B (Proc. Suppl.) 48 (1996). Paper astro-ph/9601090 .
[3] L. Gonzalez-Mestres, "Superluminal matter and high-energy cosmic rays", May 1996 . Paper astro-ph/9606054 .
[4] L. Gonzalez-Mestres, "Physics, cosmology and experimental signatures of a possible new class of superluminal particles", to be published in the Proceedings of the International Workshop on the Identification of Dark Matter, Sheffield (England, United Kingdom), September 1996 . Paper astro-ph/9610089 .
[5] L. Gonzalez-Mestres, "Physical and cosmological implications of a possible class of particles able to travel faster than light", contribution to the 28^{th} International Conference on High-Energy Physics, Warsaw July 1996 . Paper hep-ph/9610474 .
[6] See, for instance, L. Bergstrom et al., "THE AMANDA EXPERIMENT: status and prospects for indirect Dark Matter detection", same Proceedings as for ref. [4]. Paper astro-ph/9612122 .
[7] See, for instance, P.J.E. Peebles, "Principles of Physical Cosmology", Princeton Series in Physics, Princeton University Press 1993 .
[8] S.S. Schweber, "An Introduction to Relativistic Quantum Field Theory", Row, Peterson and Company 1961 .
[9] See, for instance, R.F. Streater and A.S. Wightman, "PCT, Spin and Statistics, and All That", Benjamin, New York 1964 ; R. Jost, "The General Theory of Quantized Fields", AMS, Providence 1965 .
[10] C. Itzykson and J.B. Zuber, "Quantum Field Theory", McGraw-Hill 1985 .
[11] The relativity principle was formulated by H. Poincaré, Speech at the St. Louis International Exposition of 1904 , The Monist 15 , 1 (1905).
[12] See, for instance, "Tachyons, Monopoles and Related Topics", Ed. by E. Recami, North-Holland 1978 , and references therein.
[13] See, for instance, E. Recami in [10] .
[14] See, for instance, E.C.G. Sudarshan in [10] .